Math 55 Discussion problems 30 Mar

1. What is the variance of the number of times a 6 appears when a fair die is rolled 10 times?
2. Show that if X and Y are independent random variables, then $V(XY) = E(X)^2V(Y) + E(Y)^2V(X) + V(X)V(Y)$.
3. Use Chebyshev's inequality to find an upper bound on the probability that the number of tails that come up when a biased coin with probability of heads equal to 0.6 is tossed n times deviates from the mean by more than $\sqrt{n}$.
4. Suppose that a fair octahedral die and a fair dodecahedral die are rolled together.
   (a) What is the expected value of the sum of the numbers that come up?
   (b) What is the variance of the sum of the numbers that come up?
5. Use Chebyshev's inequality to show that the probability that more than 10 people get the correct hat back when a hatcheck person returns hats at random does not exceed $\frac{1}{100}$ no matter how many people check their hats.
6. The covariance of two random variables X and Y on a sample space S, denoted by $\mathrm{Cov}(X, Y)$, is defined to be the expected value of the random variable $(X - E(X))(Y - E(Y))$. That is, $\mathrm{Cov}(X, Y) = E((X - E(X))(Y - E(Y)))$. Show that $\mathrm{Cov}(X, Y) = E(XY) - E(X)E(Y)$, and use this result to conclude that $\mathrm{Cov}(X, Y) = 0$ if X and Y are independent random variables.
7. Find $\mathrm{Cov}(X, Y)$ if X and Y are the random variables with $X((i, j)) = 2i$ and $Y((i, j)) = i + j$, where i and j are the numbers that appear on the first and second of two dice when they are rolled.